

Intuitionistic Description Logic for Legal Reasoning ^{*}

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Abstract

Classical Logic has been frequently used as a basis for knowledge representation and reasoning in many specific domains. Legal Knowledge Representation is particularly interesting due to the natural occurrences of conflicts among law systems, individual laws and cases. Those conflicts are usually taken as logical inconsistencies. Due to its inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in most other domains. An adequate intuitionistic semantics for negation in a legal domain comes to the fore when we take legally valid individual statements as the inhabitants of our legal ontology. This allows us to elegantly deal with particular situations of legal coherence, such as conflict of laws, as those solved by Private International Law analysis. This paper: (1) Presents our version of Intuitionistic Description Logic, called iALC for Intuitionistic ALC (ALC being the canonical classical description logic system) (2) Shows a study of the logical coherence analysis of “Conflict of Laws in Space”, in the scope of Private International Law, by means of the iALC Sequent Calculus proposed. (3) Discusses the relationship between ours and other logical approaches to Legal Reasoning.

Keywords: Private International Law, Intuitionistic Logic, Description Logic, Sequent Calculus.

1 Introduction

The article proposes another logical basis for *Legal Reasoning* (LR) in the context of AI. There are many aspects to be considered whenever we pose a foundational question such as the investigation of the logical basis for representing LR. Each *aspect* is related to what we want to convey in the reasoning. For example, if we want merely to classify the laws of a country in terms of jurisdiction, we only need to show, for each law,

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where it was edited: county, state, federal union, etc. On the other hand, if we want to explain the rationale a judge followed in order to reach a particular sentence, a much more complex *Knowledge Representation* (KR) is needed to support LR. In the former situation, the mere list of law editions, for each jurisdiction, indexed by the “laws” themselves, would serve as a “representation” system, while regarding the latter, the situation is far more complex. It seems clear that the way LR can be conveyed is strongly related to the way laws (or “the law”) are represented. That is, LR is strongly interconnected to Legal Knowledge Representation, i.e, the Legal Ontology chosen, in a wider sense of the term *Ontology*. What we see is that LR must have *ontological commitments*. It cannot be only based on logic.

In [5], KR is taken as a kind of *surrogate*, a substitute of the represented knowledge itself. Their authors state some fundamental roles that KR plays: (1) It is a set of *ontological commitments*; (2) It is a fragmentary theory of intelligent reasoning; (3) It is a medium for *efficient computation*; and finally (4) It is a medium of *human expression*. When our approach is analyzed according to these roles, we can summarize the analysis in two principles:

1. LR should be based on logic. This principle is supported by the fact that we focus on *reasoning* rather than on learning or data-mining. Our main purpose is to have a basis to generate human readable explanations of why a statement is a valid legal statement with respect to some legal system.
2. The ontological commitments of LR should be guided by the underlying jurisprudence theory or by judicial practice, but not by both. Also the ontological commitments need to pay attention to computational efficiency and to the feasibility of the reasoning.

When dealing with KR, questions about the adequacy of the *surrogate* come to the fore. The *surrogate* must substitute the “real thing” as faithfully as possible, as far as the main purpose of its own design is concerned. Taking into account the logical approach (principle 1), there cannot be inconsistencies in the set of sentences describing the knowledge. An inconsistent theory has no model, and hence, describes/represents nothing. Inconsistency is for most of the known logics, a concept strongly related to negation. A theory is inconsistent, if and only if, it contains a sentence “*A*” and a sentence “*not A*”. The concept of logical negation also comes to the fore when representing legal knowledge. Consistency seems to be more difficult to maintain when more than one coherent *law system* can judge a case. This is also called *conflict of laws*. There are some legal mechanisms to solve these conflicts, some of them state privileged fori, others rule jurisdiction. In most of the cases, the conflict is solved by admitting a law hierarchy or precedence. Even using these legal mechanisms, coherence is still a main concern in legal systems. As consistency is a direct consequence of logical negation, its logical basis has to be more subtle than it is in other domains.

Description Logics (DL) have been widely used as a general logical basis for KR. This use relies on two basic features:

- DL are basically conceptual languages. A KR based in a DL is designed on top of a set of concept terms, a set of binary-relations (the TBOX), some statements

about individuals (the ABOX), and a set of subsumption assertions between concepts. Entity Relationship models, UML specifications, and many other frameworks in Computer Science can be adequately represented by DL theories.

- DLs are decidable. There are good (state-of-the-art) algorithms to test satisfaction (truth) of a DL formula, and these algorithms can be used to test logical consequence of subsumptions with respect to a theory. In this article, following our principle 2, we choose to build LR on top of the tools and methods already known from the DL community. Some of our previous work on the generation of explanation for DL-based reasoning is also one of our motivations for using DL in LR. In section 3, we see that our underlining jurisprudence theory requires the use of an intuitionistic version of ALC (the core logic of DLs). The tailoring of a intuitionistic version of ALC (iALC) is also a contribution of this work.

It is worth explaining to a modal logician that ALC is formally equivalent to multi-modal K [22]. Under the modal logic point of view, ALC with ABOXes can be taken as a hybrid logic. The assertion $i: Concept$, expressing that i is an individual inhabiting the extension of the $Concept$ ¹ is interpreted as “the proposition $Concept$ holds at world i ”, according to the formal relationship between ALC and modal logic. The reader is advised to keep both readings of ALC in mind to understand our definition of iALC and its use in LR. In what follows, each individual law will be taken as a possible world (in a Kripke structure) and the collection of all individual laws (valid legal statements) forms our Legal World (the real one, the thing that is represented).

Given the existent and proposed applications of the Semantic Web, there has been a fair amount of work into finding the most well-behaved system of description logic that has the broadest application. As discussed in [6], considering versions of *constructive* description logics makes sense, both from a theoretical and from a practical viewpoint. There are several possible and sensible ways of defining *constructive* description logics. Here we use a constructive version of ALC, based on the framework for constructive modal logics developed by Simpson in his PhD thesis [23], instead of e.g. [15], to the problem of formalizing Legal Knowledge. We choose “Conflict of Laws in Space”, to show how intuitionistic negation (in iALC) deals with incoherence.

Our system is defined along the lines of the constructive framework developed by Brauner and de Paiva in [4] for Hybrid Logics. We reason that the existence of frameworks for constructive modal and constructive hybrid logics in the labelled style of Simpson, hint at a good style of constructive description logics, in terms of both worked out foundations and ease of implementation. The use of nominals, individuals in the description logic terminology, is taken for granted.

Some preliminary and shorter versions of this work was presented at the JURIX 2010 [9] by the first author, Hylo 2010 [7] and Loait 2010 [10] by the third author.

2 Related Work

In this section we briefly discuss two other main alternative approaches to LR: (1) The Deontic Logic approach, and, (2) The Defeasible Logic approach.

¹A concept extension in ALC is merely a set.

Of course, we could not discuss all the literature from deontic logic in Law. Since von Wright’s seminal paper [25] ² many approaches to LR using deontic modalities have been proposed. While the motivation is clear, deontic reasoning seems to have almost the same problems as their non-deontic counter-parts. The many paradoxes that are known from philosophical logic, concerning moral and/or legal questions, are not solved with deontic logic. In fact, they are only concisely expressed in deontic logic. The *Good Samaritan* paradox, for example, is obtained by the mere use of the axiom K of the deontic logic KD . Many contrary-to-duty paradoxes, including some based on existing laws of actual countries (see [17]), are expressed, but they are not solved under the simply deontic setting. Alchourrón [1] has proposed, still under deontic logic, a semantic approach (*Logic without Truth*) that aims to solve many of these paradoxes. Alchourrón suggests that a norm should not always have assigned truth values. We agree with this, but our approach is distinct, since for us a norm never has a truth value. In this way we are closer to Kelsen’s dichotomy (valid law vs. invalid law) than to [1]. It is important to note that the pure use of a deontic logic has been shown to be inadequate to accomplish this task. In [24] it is shown that deontic logic does not properly distinguish between the normative status of a situation and the normative status of a norm (rule).

Sartor has a well-developed approach for LR, initially described in [21], where each law is taken as a defeasible rule (or sentence). Thus, a law has the general form $\alpha \leftarrow \beta_1 \wedge \dots \wedge \beta_n \wedge S(\neg\gamma_1) \dots \wedge S(\neg\gamma_k)$, where α is the conclusion, each β_i is a *principal fact*, and each γ_j is a *secondary fact*. In a trial, the onus of the proof of a principal fact is on the *plaintiff*, while the onus of the proof of any of the γ_j , i.e., a secondary fact, is on the *defendant*. The law can be applied, if and only if, the defendant cannot provide proofs for the secondary facts. Terms like *constitutive* and *impediment* are also used instead of primary and secondary facts, respectively. We can say that Sartor’s approach is dynamic, it aims at showing how a law is or can be used in a trial. Each law application corresponds to a defeasible rule application at the logical level. There are other approaches to LR using defeasible reasoning, they are not much distinct, in essence, from [21]. Comparing these approaches to ours, we can say that ours is static. We represent the Legal World only by means of the laws, facts are not laws. In our setting the reasoning is conveyed as if the legal proofs admitted at the trial were already laws (individually valid legal statements). We can express, as does [21], the reason why the judge provided the sentence, but we cannot provide any clues about the dynamics of the trial. On the other hand, since we do not use defeasible reasoning, our approach is computationally and logically simpler. We could say, on the basis of principle 2, that Sartor’s approach is based/committed to Judicial Practice while ours is based/committed to jurisprudence theory.

3 Jurisprudence and Intuitionism

Here we examine the adherence of our approach to the principle 2. One of the main problems from jurisprudence (legal theory) is to make precise the use of the term “law”.

²Mally [14] was in fact the first to envisage a logic for norms

In fact, the problem of individuation, namely, what counts as the unit of law, seems to be one of the fundamental open questions in jurisprudence. Any approach to law classification requires firstly answering the question “What is to count as one complete law?” ([20]). There are two main approaches to this question. One is to take all (existing) legally valid statements as a whole as the law. This totality is called “the law”. This approach is predominant in legal philosophy and jurisprudence and owes its significance to the Legal Positivism tradition initiated by Hans Kelsen (for a contemporary reference see [13]). The coherence of “the law” plays a central role in this approach. The debate on whether coherence is built-in by the restrictions induced by Nature in an evolutionary way, or whether coherence should be the object of knowledge management, seems to be classical one. The other approach to law definition is to take all legally valid statements as being *individual laws*. This view, in essence, is harder to be shared with jurisprudence principles, since they are mainly concerned in justifying the law. This latter approach seems to be more suitable to Legal AI. It is also considered by legal theoreticians, at least partially, whenever they start considering ontological commitments, such as, taking some legal relations as primitive ones [12], *primary and secondary rule* [11] or even a two-level logic to deal with different aspects of law (see *logic-of-imperation/logic-of-obligation* from [3]). In fact, some Knowledge Engineering groups pursue this approach as a basis for defining legal ontologies. We also follow this route. In what follows we shall use VLS to denote the expression “valid legal statement”.

In the sequel we discuss the role of the negation in Legal Knowledge Representation. Remember that we will take valid legal statements as individuals of a legal universe, instead of taking them as (deontic) propositions. According to this approach, the natural precedence between valid legal statements is ruled by the pre-order relationship that characterizes the Kripke Semantics for Intuitionistic Logic. In order to find out what happens in (classical) ALC, we can verify the simple task of negating an ALC concept. We remind the reader that in ALC the formula $i: C \sqcup i: \neg C$ is valid, and hence a theorem, for any nominal i and concept C .

The validity of the above formula, for arbitrary i and C , holds only in a classical logic setting. Consider that Peter, a young man, is not yet 18 years old. Let us write BR for the set of all VLS in Brazil. If pl is the VLS for “Peter is liable” then $pl: BR$ cannot be the case, since the legal age in Brazil is 18. However, we know that $pl: BR \sqcup pl: \neg BR$ and from $pl: BR$ not being the case, we are forced to admit $pl: \neg BR$. This conclusion is too strong, it says that “Peter is liable” is a VLS outside Brazil. Nobody can ensure that, there might be no Peter at all, outside Brazil. Classical ALC is not adequate for representing LR according to our jurisprudence foundation. On the other hand, if we consider an intuitionistic version of ALC, we do not have to admit that $pl: \neg BR$. The semantics of the negation in intuitionistic logic takes into account possible worlds, as in modal logic, and an hereditary relationship between worlds. In terms of intuitionistic semantics, $pl: \neg BR$ holds whenever every VLS accessible³ to pl does not belong to BR . Here we can see the second reason to use intuitionistic logic as a basis to LR. The relationship between VLSs is the natural precedence existing between laws. For example, the VLS “Peter is president of a Com-

³A world a is accessible from world b if a is related by means of the hereditary relationship to b .

pany” must be preceded by pl , no one can be president of a company without being liable. According to this reading the proposition $pl: \neg BR$, only takes into account VLS that succeed pl in order to find out whether “Peter is liable” does belong to $\neg BR$. In other worlds, pl must exist in any case, and cannot be arbitrarily a valid statement abroad Brazil, on the basis it is not a VLS in Brazil. Our conclusion is that intuitionistic negation deals with classification of concepts better than classical negation whenever one has a natural precedence between VLS.

From the semantic point of view, iALC seems to be well suited to model the legal theoretic approach pursued by KR as considered above. Let us consider an iALC model having as individuals each of the valid and possible *legal statements*. The \preceq relation is the natural hierarchy existing between these *legal statements*, as well from any precedence relation related to them. For example, sometime conflicts between *legal statements* are solved by inspecting the age of the laws (what is the date of its first edition in the legal system), the wideness enforcement scope of each law, etc. Any of these relations are order relations. For example, “Theodor is vicariously liable by John” is legally dominated (precedes) by “John is a *worker* of Theodor”, or “John and Theodor have an *employment contract*”. Any legal statement involving the *civil liability* of someone must be preceded by an legal statement asserting that he/she is of *legal age*. If C is a concept symbol in a description logic language, its semantics is the subset of *legal statements* representing a *kind of* legal situation.

We can say that our logical approach to LR has a strong bias to static analyses of the Legal World. Cases that are not yet sentenced cannot be taken into account. However, VLS related to the case, as proofs that are legally part of a trial, can be considered.

The main role of Intuitionism in this setting is to provide semantics to negation of concepts, as well as to subsumption. The iALC approach depicted in this paper basically consists of a model which includes all the possible valid legal statements and the relations among them induced by the natural precedence between VLS.

4 The system iALC

Traditionally modal logics are classical propositional logics augmented with modalities for necessity, possibility, obligations, provability, etc. While by no means the most popular ones, there are also several reasonable systems of *constructive* modal logics in the literature. In this paper we are mostly concerned with the framework proposed by Simpson [23] and called IML (intuitionistic modal logic). This consists of a series of constructive Natural Deduction systems, which arise from interpreting the usual possible worlds definitions in an intuitionistic meta-theory. The different systems correspond to different kinds of axioms satisfied/different conditions asserted of the accessibility relations. The main benefit of these Natural Deduction systems when compared to traditional axiomatizations is their susceptibility to proof-theoretic techniques. Strong normalization and confluence results are proved for all of the systems described by Simpson. On the downside the basic structure of Natural Deduction needs to be extended to deal with assumptions of the form the world x is R -related to world y , which is written as a second kind of formula xRy .

Building up from Simpson’s framework for constructive modal logics, Braüner and

de Paiva introduced *intuitionistic hybrid logics*, denoted by IHL, in [4]. Hybrid logics add to usual modal logics a new kind of propositional symbols, the nominals, and also the so-called satisfaction operators. A nominal is assumed to be true at exactly one world, so a nominal can be considered the name of a world. If x is a nominal and X is an arbitrary formula, then a new formula $x : X$ called a satisfaction statement can be formed. The part $x :$ of $x : X$ is called a satisfaction operator. The satisfaction statement $x : X$ expresses the fact that the formula X is true at one particular world, namely the world at which the nominal x is true. Out of these tightly connected systems of intuitionistic modal IML and hybrid logics IHL, we want to carve out our system of intuitionistic description logic iALC.

The system iALC is a basic description language [2]. Its concept formers are described by the following grammar:

$$C, D ::= A \mid \perp \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

where A stands for an atomic concept and R for an atomic role. This syntax is more general than standard ALC in that it includes subsumption \sqsubseteq as a concept-forming operator. Negation can be represented via subsumption, $\neg C = C \sqsubseteq \perp$, but we find it convenient to keep it in the language. The constant \top can also be omitted since it can be represented by $\neg\perp$.

A constructive interpretation of iALC is a structure \mathcal{I} consisting of a non-empty set $\Delta^{\mathcal{I}}$ of entities in which each entity represents a partially defined individual; a refinement pre-ordering $\preceq^{\mathcal{I}}$ on $\Delta^{\mathcal{I}}$, i.e., a reflexive and transitive relation; and an interpretation function $\cdot^{\mathcal{I}}$ mapping each role name R to a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and each atomic concept A to a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ which is closed under refinement, i.e., $x \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $y \in A^{\mathcal{I}}$. The interpretation \mathcal{I} is lifted from atomic \perp, A to arbitrary concepts via:

$$\begin{aligned} \top^{\mathcal{I}} &=_{df} \Delta^{\mathcal{I}} \\ (\neg C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\ (C \sqcap D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\ (C \sqcup D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\ (C \sqsubseteq D)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. (x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\ (\exists R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow \exists z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \text{ and } z \in C^{\mathcal{I}}\} \\ (\forall R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow \forall z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\} \end{aligned}$$

Our setting is a simplification of Mendler and Scheele's [15] where we dispense with infallible entities, since our system iALC satisfies (like classical ALC) $\exists R.\perp = \perp$ and $\exists R.(C \sqcup D) = \exists R.C \sqcup \exists R.D$. We will have no use for nested subsumptions, but they do make the system easier to define, so we keep the general rules.

Simpson's original system is a Natural Deduction (ND) system meant to capture in the rules exactly the intuitions of the modalities over possible worlds. A sequent calculus that corresponds to Simpson's ND is discussed by Negri [16].

A sequent calculus for iALC, based on the classical one first presented in [19] is presented in [9]. The calculus in [19] uses labels for context controlling. This control

avoids unsound uses of the propositional rules, e.g, by not allowing that one can derive $\Delta \Rightarrow \exists R(\alpha \sqcup \beta)$ from $\Delta \Rightarrow \exists R\alpha$ and $\Delta \Rightarrow \exists R\beta$. The system described in [19] is an intermediate step towards the Natural Deduction (ND) system, presented in another article. In ND each concept derivation is regarded to a sequence of roles, those that were focus of an elimination rule. Using labels we can define a \forall -Elim as $L\forall R.\alpha \vdash L,\forall R\alpha$. Thus, $\forall R_1,\forall R_2\alpha$ also means $\forall R_1\forall R_2.\alpha$. This is the main purpose of labeling concepts in our previous systems. The labels can also be used to provide direct means to derive $\Delta \Rightarrow L(\forall R.\alpha)L_2$ from $\Delta \Rightarrow L,\forall R\alpha$, that is, to derive a concept inside the context of a list of roles. The labels are roles and $\forall R_1,\exists R_2\alpha$ is a labeled formula representing $\forall R_1\exists R_2.\alpha$. This mechanism is shown (see [18]) to be quite useful when defining a Natural Deduction system for Description Logic, in particular ALC. For iALC we have already labels for the individuals, so in this article the traditional version, without labels for roles is presented. The system is shown in Figure 1.

$\frac{\Delta_1 \Rightarrow \alpha \quad \Delta_2, \beta \Rightarrow \gamma}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \gamma} \sqsubseteq\text{-l}$	$\frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta} \sqsubseteq\text{-r}$
$\frac{\Delta, \alpha, \beta \Rightarrow \gamma}{\Delta, \alpha \sqcap \beta \Rightarrow \gamma} \sqcap\text{-l}$	$\frac{\Delta \Rightarrow \alpha \quad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta} \sqcap\text{-r}$
$\frac{\Delta, \alpha \Rightarrow \gamma \quad \Delta, \beta \Rightarrow \gamma}{\Delta, \alpha \sqcup \beta \Rightarrow \gamma} \sqcup\text{-l}$	
$\frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta} \sqcup\text{-l-r}$	$\frac{\Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcup \beta} \sqcup\text{-r}$
$\frac{\delta \Rightarrow \gamma}{\exists R.\delta \Rightarrow \exists R.\gamma} \text{prom-}\exists$	$\frac{\Delta \Rightarrow \gamma}{\forall R.\Delta \Rightarrow \forall R.\gamma} \text{prom-}\forall$

Figure 1: The System iALC: logical rules

5 Conclusions

In this article we used iALC, a constructive description logic, to provide an alternative, and, we claim, more adequate, definition for subsumption in LR domain. The system iALC deals with the jurisprudence theory that considers “The Law” as all (possible) legally valid individuals laws. Conflict of laws is formalized by means of iALC, which shows the latter adequacy to perform coherence analysis in legal AI. The sequent calculus provides this formalization. Our approach is different from [8], where the reasoning

is by means of argumentation and opposition of propositions. Our basis is quite distinct, since we do not consider “laws” as propositions. Our approach is static while Sartor’s approach is dynamic, but this also means that our logical basis is simpler, and consequently, computational support should be easier.

Our sequent calculus first appeared in Hyla 2010 Workshop [7]. The use of sequent calculus proofs as a basis for argumentation and (human-readable) generation of proofs is one of the main goals of the authors in general and in the domain of Law formalization and Representation.

Finally, as far as the contrary-to-duty and other deontic paradoxes are concerned, we do not think that the logical basis proposed in this article has better results than Alchourrón’s *Logic without Truth* [1] approach. We leave it to future work to obtain a better understanding of the comparison of results.

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