Intuitionistic Logic and Legal Ontologies

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Abstract. This paper briefly shows how Intuitionistic Description Logic can be considered a good alternative to classical ALC as far as formalizing legal knowledge is concerned.

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1. Introduction

The concept of logical negation comes to the fore when representing legal knowledge. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one *law system* can judge a case. This is called a *conflict of laws*. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, others ruling jurisdiction, etc. In most of the cases, the conflict is solved by admiting a law hierarchy or a law precedence. Even using these mechanisms, coherence is still a major issue in legal systems. Each layer in this legal hierarchy has to be consistent. As consistency is a direct consequence of how one deals with logical negation, negation is also a main concern of legal systems.

Here we briefly present iALC, an intuitionistic version of the classical description logic ALC, based on the framework for constructive modal logics presented by Simpson([Simpson1995]) and adapted to description languages by [dePaiva2003]. We apply this logic to the problem of formalizing legal knowledge. In order to illustrate our approach, we formalize "Conflict of Laws in Space". This conflict happens when several laws can be applied, with different outcomes, to a case/situation depending on the place where the case occurs. Typical examples are those ruling the rights of a citizen abroad.

2. Jurisprudence and Intuitionism

One of the main problems in Jurisprudence is to make precise the meaning of term "law". In fact, the problem of individuation, namely, of deciding what counts as the unit of law, seems to be one of the fundamental open questions in jurisprudence. Any approach to law classification requires first answering the question "What is to count as one complete law?" ([Raz1972]). There are two main approaches to this question. One is to take all existing legally valid statements as a whole as one law. This totality is called "the law". This approach is predominant in legal philosophy and jurisprudence. It owes its significance to the Legal Positivism tradition initiated by Hans Kelsen (for a contemporary reference see [Kel91]). The coherence of "the law" plays a central role in this approach, raising a debate on whether coherence is built-in by the restrictions induced by Nature in an evolutionary way, or whether it should be the object of knowledge management. The other approach to law definition is to consider all legally valid statements as being *individual laws*. This view is harder to be associated with justifying the law. However, it seems to be more suitable to Legal AI. It also considers taking some legal relations as primitive (Hohfeld, 1919), *primary and secondary rule* (Hart, 1961) or even a logic to deal with different aspects of law (see *logic-of-imperation/logic-of-obligation* from Bentham, 1970). We adopt this latter approach in formalizing legal knowledge. It is important to note that we avoid the use of any deontic logic, since this leads to well-known *contrary-to-duty* paradoxes and their variants. Besides "laws" are socially motivated mechanisms and should be not considered in terms of truth. [Valente1995] discusses why deontic logic does not properly distinguish between the normative status of a situation and the normative status of a norm (rule).

Our approach will take *legal statements* as individuals of a legal universe, instead of propositions. The latter will be used to classify the *legal statements*. The natural (socially established) precedence between individual *legal statements* will be associated to the pre-order relationship that characterizes the Kripke Semantics for Intuitionistic Logic **IL**. In this article we use the short form **VLS** to denote a *valid legal statement*.

In order to compare our approach to the classical one, we can use the simple task of negating a legal statement. Consider the proposition "Peter is liable", and assume that Peter is under the legal age in Brazil. The **VLS** representing this situation, in classical logic, is obtained by negating the proposition. "Peter is not liable" is the **VLS** inhabiting the world, which belongs to the Brazilian collection of **VLS**. Classical logic, validating the *non tertius datur* principle (the excluded third principle says that$\alpha \vee \neg\alpha$ holds always), classifies the negation of "Peter is liable" as "Peter is not liable". Taking intuitionistic logic **IL**, one may have neither "Peter is liable" nor "Peter is not liable" as part of the Brazilian collection of **VLS**. The semantics of **IL** avoids classification by negating a proposition. Thus, using **IL**, it is possible to consistently have a legal situation where Peter is taken as liable, even if he is under the legal age, for example. For the classical case, *not being liable* is the same that *being not liable*.

From the semantic point of view, iALC seems well suited to modeling the legal theoretic approach pursued by knowledge engineering, as described above. An iALC model has as individuals (Kripke worlds) the **VLS**s. The $\preceq$ relation is the natural hierarchy existing between these individual legal statements, as well as any precedence relation associated to them. For example, sometimes conflicts between legal statements are solved by inspecting the age of the laws (what is the date of the law's first edition in the legal system?), the wideness of enforcement scope of each law, etc. Any of these relations can be considered an order relation. For example, *"Theodor is vicariously liable by John"* is legally dominated (preceded) by *"John is a worker of Theodor"*, or *"John and Theodor have an employment contract*. Any legal statement involving the *civil liability* of someone must be preceded by any legal statement asserting that this person is of *legal age*.

3. The system iALC

iALC is a description language with nominals. Its concept formers are described by the grammar:

$$C, D ::= A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

where A, R stand for atomic concept and role. This syntax is more general than standard ALC for it includes subsumption $\sqsubseteq$ as a concept-forming operator. We do not use nested subsumptions, but they make the system easier to define. Negation can be represented via subsumption, $\neg C = C \sqsubseteq \bot$, but we find it convenient to keep it in the language. The constant $\top$ can also be omitted since it can be represented by $\neg\bot$. To this language we add nominals, which 'name' the world where the concepts hold. Thus, $x : C$ and $x \preceq y$ are formulas, where x and y are nominals. An interpretation of iALC is a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \preceq^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a non-empty set of entities VLS's; $\preceq^{\mathcal{I}}$ is a given pre-ordering on $\Delta^{\mathcal{I}}$; and $\cdot^{\mathcal{I}}$ is a mapping of role names R to $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and atomic concepts A to $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, (satisfying $x \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $y \in A^{\mathcal{I}}$), such that:

$$
\begin{aligned}
\top^{\mathcal{I}} &=_{df} \Delta^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. (x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow \exists z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \text{ and } z \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow \forall z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{aligned}
$$

Simpson's original system is a constructive Natural Deduction system whose rules capture exactly the intuitions of the modalities over possible worlds. We simply adapt it to the description logic fragment. The rules are in Figure 1 in the appendix. The labels intuitively describe the world where the concept is being asserted. Thus $x : C$ means that the **VLS** x is in C. (In terms of Kripke models, $\models_x C$, where x is a world). Our deductive system also has assertions of the form xRy, meaning that the role R relates **VLS**s x and y.

4. Private International Law using iALC

A concept symbol C, in a description logic language, is associated to the subset of **VLS**s representing a *kind* of legal situation. Roles in the description logic language are associated to relations between these *legal situations*, imposed by the relationship between each pair of individual *legal statements*. In order to prove that a **VLS** belongs to a legal situation, one may consider some of the **VLS**s that it precedes also in the concept. This is represented by the framed sequent in the proof below. Consider the following situation: "*Peter and Maria signed a renting contract. The contract concerns an apartment in Rio* *de Janeiro. It states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.* "

Write $\mathtt{BR}$ for the collection of legal statements valid in Brazil, $\mathtt{SC}$ for the ones valid in Scotland, and cmp for the syntactic counterpart of *Peter and Maria have contractual...*, we will prove that the **VLS** cmp is in $\mathtt{BR}$, $comp : \mathtt{BR}$. The statement *Maria is liable*, which we write as ml, is in $\mathtt{BR}$. Similarly, pl for *Peter is liable* is in $\mathtt{SC}$. Only liable people have civil obligations. Thus, $cmp \preceq pl$ and $cmp \preceq ml$. If we write $\mathtt{PIL}$ for *Private International Law* statements that are in $\mathtt{BR}$, then $\mathtt{PIL} \sqsubseteq \mathtt{BR}$. $\mathtt{PIL}$ is a disjunction of concepts of legal statements including $\exists\mathtt{LexDom.ABROAD}$. $\mathtt{PIL}$ relates legal statements in different contexts, locations, times, etc. Each member of $\mathtt{PIL}$ is concerned with a context, in particular $\mathtt{ABROAD}$, $\mathtt{AB}$ in the proof, is the union of the **VLS**s holding in all countries, except Brazil. *LexDomicilium* is a legal connection, a relation between laws, $\mathtt{LexDom}$ in the proof. The pair $\langle pl, pl\rangle$ is in $\mathtt{LexDom}$, for Peter lives in Scotland, abroad as far as Brazil is concerned. Using Δ and Ω below we will prove that $cmp : \mathtt{BR}$.

$$\boxed{\begin{array}{l}\Delta = \{ml : \mathtt{BR}, pl : \mathtt{SC}, cmp \preceq pl, cmp \preceq ml, pl\ \mathtt{LexDom}\ pl\} \\ \Omega = \{\mathtt{PIL} \Rightarrow \mathtt{BR}, \mathtt{SC} \Rightarrow \mathtt{ABROAD}, \ldots \sqcup \exists\mathtt{LexDom.ABROAD} \sqcup \ldots \Rightarrow \mathtt{PIL}\}\end{array}}$$

Call Π the following proof and use it in the second derivation below:

$$\dfrac{\dfrac{\dfrac{\Delta \Rightarrow pl : \mathtt{SC}}{\Delta \Rightarrow pl : \mathtt{AB}}\sqcup\text{-R} \quad \Delta \Rightarrow pl\ \mathtt{LexDom}\ pl}{\Delta \Rightarrow pl : \exists\mathtt{LexDom.AB}}\exists\text{-R} \qquad \dfrac{\dfrac{\exists\mathtt{LexDom.AB} \Rightarrow \exists\mathtt{LexDom.AB}}{\exists\mathtt{LexDom.AB} \Rightarrow \mathtt{PIL}}\sqcup\text{-R} \quad \mathtt{PIL} \Rightarrow \mathtt{BR}}{\exists\mathtt{LexDom.AB} \Rightarrow \mathtt{BR}}\,cut}{\Delta \Rightarrow pl : \mathtt{BR}}\text{inc-R}$$

$$\dfrac{\Delta \Rightarrow ml : \mathtt{BR} \qquad \dfrac{\dfrac{\Pi}{\Delta \Rightarrow pl : \mathtt{BR}} \quad \dfrac{}{ml : \mathtt{BR}, pl : \mathtt{BR} \Rightarrow cmp : \mathtt{BR}}}{\Delta, ml : \mathtt{BR} \Rightarrow cmp : \mathtt{BR}}\text{cut}}{\Delta \Rightarrow cmp : \mathtt{BR}}\text{cut}$$

5. Conclusions

The system iALC is used to provide an alternative definition for subsumption that copes with the theory that considers "The Law" as all (possible) legally valid individuals statements. Conflict of laws can be formalized in iALC and shows the adequacy of the approach to perform coherence analysis in legal AI. In [DungSartor2010] , reasoning is done by means of argumentation and opposition of propositions. We do not consider "laws" as propositions, hence our approach is distinct. In future work we will discuss how we adhere to the main principles stated in Sartor's book[DungSartor2010].

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A. Appendix

The rules of the system iALC are as follows:

$$\frac{}{\Gamma, x\colon C \Rightarrow\ x\colon C, \Delta} \qquad \frac{}{xRy, \Gamma \Rightarrow\ \Delta, xRy}$$

$$\frac{C_1 \Rightarrow\ C_2 \qquad x : C_2 \Rightarrow\ \Delta}{x : C_1 \Rightarrow\ \Delta}\ \text{inc-l} \qquad \frac{\Gamma \Rightarrow\ x : C_1 \qquad C_1 \Rightarrow\ C_2}{\Gamma \Rightarrow\ x : C_2}\ \text{inc-R}$$

$$\frac{\Gamma_1 \Rightarrow\ C \qquad \Gamma_2, D \Rightarrow\ \Delta}{\Gamma_1, \Gamma_2, C \sqsubseteq D \Rightarrow\ \Delta}\ \sqsubseteq\text{-l} \qquad \frac{\Gamma, C \Rightarrow\ D}{\Gamma \Rightarrow\ C \sqsubseteq D}\ \sqsubseteq\text{-r}$$

$$\frac{\Gamma, x\colon C, x\colon D \Rightarrow\ \Delta}{\Gamma, x\colon (C \sqcap D) \Rightarrow\ \Delta}\ \sqcap\text{-l} \qquad \frac{\Gamma \Rightarrow\ x\colon C,\Delta \qquad \Gamma \Rightarrow\ x\colon D,\Delta}{\Gamma \Rightarrow\ x\colon (C \sqcap D),\Delta}\ \sqcap\text{-r}$$

$$\frac{\Gamma, x\colon C \Rightarrow\ \Delta \qquad \Gamma, x\colon D \Rightarrow\ \Delta}{\Gamma, x\colon (C \sqcup D), \Rightarrow\ \Delta}\ \sqcup\text{-l} \qquad \frac{\Gamma \Rightarrow\ x\colon C, x\colon D, \Delta}{\Gamma \Rightarrow\ x\colon (C \sqcup D), \Delta}\ \sqcup\text{-r}$$

$$\frac{\Gamma, x\colon \forall R.C, y\colon C, xRy \Rightarrow\ \Delta}{\Gamma, x\colon \forall R.C, xRy \Rightarrow\ \Delta}\ \forall\text{-l} \qquad \frac{\Gamma, xRy \Rightarrow\ y\colon C, \Delta}{\Gamma \Rightarrow\ x\colon \forall R.C, \Delta}\ \forall\text{-r}$$

$$\frac{\Gamma, xRy, y\colon C \Rightarrow\ \Delta}{\Gamma, x\colon \exists R.C \Rightarrow\ \Delta}\ \exists\text{-l} \qquad \frac{\Gamma \Rightarrow\ \Delta, xRy \qquad \Gamma \Rightarrow\ \Delta, y\colon C}{\Gamma \Rightarrow\ \Delta, x\colon \exists R.C}\ \exists\text{-r}$$

Figure 1. iALC Rules